MODELING AND SIMULATION OF VSC-HVDC WITH DYNAMIC PHASORS

Wei YAO¹, Jinyu WEN², Shijie CHENG³

¹,²,³Electric Power Security and High Efficiency Lab, Huazhong University of Science and Technology, Wuhan 430074, China

¹E-mail: yao_wei@163.com  ²E-mail: jinyu.wen@hust.edu.cn  ³E-mail: sjcheng@mail.hust.edu.cn

ABSTRACT

A novel dynamic phasors method to model voltage sources converter based HVDC (VSC-HVDC) transmission system is proposed in this paper. This approach is based on the time-varying Fourier series coefficients of the system variables, and focused on the dynamics of the Fourier coefficients. Followed by the time-domain converter station model for VSC-HVDC described by switch function, detailed analysis of the VSC-HVDC dynamic phasors model is presented in this paper. The VSC-HVDC model is simplified by keeping important system state variables that corresponding to the time-varying Fourier series, which include the converter station switching function considering both the DC component and basic frequency component, and the DC transmission line considering only the DC component. Therefore, high frequency switching process is greatly simplified. Typical simulation results show that this method can ensure simulation accuracy and reduce computational cost at the same time.

Keywords: VSC-HVDC; Dynamic phasors model; Time-varying Fourier coefficients; electromagnetic transient model

1. INTRODUCTION

Compared to the conventional HVDC systems, the VSC-HVDC transmission system has several advantages such as independent control of active and reactive power, dynamic voltage support at the converter bus for enhancing stability, possibility to feed to weak AC systems or even passive loads[1]. Therefore, the VSC-HVDC system can play an important role in emerging deregulated power systems and has broad prospects in the applications of future city power supply and the new energy (such as wind power generation, photovoltaic fat and small hydropower, etc.) connected to power grid[2], [3].

Many research results on principles and
Followed by the time-domain converter station model for VSC-HVDC described by switch function, detailed analysis of the VSC-HVDC dynamic phasors model is presented in this paper. The VSC-HVDC model is simplified by keeping important system state variables that corresponding to the time-varying Fourier series, which include the converter station switching function considering both the DC component and basic frequency component, and the DC transmission line considering only the DC component. Therefore, high frequency switching process is greatly simplified. The dynamic phasors model and a detailed time domain EMT model for the VSC-HVDC are simulated in MATLAB environment, and simulation results show that this method can ensure simulation accuracy and reduce computational cost at the same time.

2. OUTLINE OF THE DYNAMIC PHASORS

The method of dynamic phasors is based on the time-varying Fourier coefficients. A possibly complex time-domain waveform $x(\tau)$ can be represented on the interval $\tau \in [t-T, t]$ using a Fourier series of the form

$$x(\tau) = \sum_{k=-\infty}^{\infty} X_k(t) e^{jk\omega_0 \tau}$$

(1)

Where $\omega_0 = 2\pi/T$ and $X_k(t)$ are the Fourier coefficients and named here as dynamic phasors. The $k$th phasor at time $t$ can be determined by the following expression:

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The phasor \( X_k(t) \) are all complex quantities, which satisfy the following:

\[
\langle x \rangle_1 = \langle x \rangle_1^r + j\langle x \rangle_1^i = \langle x \rangle_{-1}^r + j\langle x \rangle_{-1}^i
\]

(3)

where the superscripts \( r \) and \( i \) denote the real and imaginary parts of the defined quantities respectively, and “*” denotes the complex conjugation.

There are two key and useful properties of the dynamic phasors:

- Differentiation of dynamic phasor: For the \( k \)th Fourier coefficient, the differential with time satisfy the following formula:

\[
\frac{dX_k(t)}{dt} = \langle \frac{dx}{dt} \rangle_k(t) - j\omega_k X_k(t)
\]

(4)

- Product of dynamic phasor: For two waveforms \( x_1(t) \) and \( x_2(t) \), the \( k \)th phasor of their product can be obtained by:

\[
\langle x_1 x_2 \rangle_k = \sum_{i=-\infty}^{\infty} \langle x_1 \rangle_{k-i} \langle x_2 \rangle_i
\]

(5)

Dynamic phasors method is based on the idea of frequency decomposition, and focus on the dynamics of the significant Fourier coefficient. By truncating unimportant higher order series and keep only those significant series, the dynamic phasors model can catch the dynamic behavior of the original detail model. A new state-space model can be obtained when we consider these reserved phasors as state variables. The model is simplified, and can keep the nonlinear of original model to large extent. Dynamic phasors can be used to model the polyphase systems under unbalanced operation including electronic converters.

The emphasis of this paper is aimed at the analysis of three phase balanced situation.

3. DYNAMIC PHASORS OF VSC-HVDC

A. The Equivalent Circuit of VSC-HVDC

Fig. 1. The structure diagram of VSC-HVDC

Fig. 1 illustrates the basic structure of VSC-HVDC. In the figure, the interface reactors (usually transformers) are applied to smooth current and secure the power exchange between AC system and DC link. The reservoir DC capacitors are used for voltage support and harmonic attenuation. The two converters on the ends of DC link have the same structure and both connect active AC networks.
Fig. 2 The equivalent circuit diagram of VSC-HVDC

In this paper, for the simplicity of analysis, we suppose that [14]:

a) The system is under balanced operating condition;
b) The converter valve are ideal and converter transformer is lossless.
c) Parameters of all the bridges of both the rectifier and the inverter are equal.

Fig.2 is the equivalent circuit diagram of VSC-HVDC. Here, $U_{S[abc]}$ and $U_{S'[abc]}$ are three phase ac infinite sources of the rectifier and of the inverter respectively; $v_{S[abc]}$ and $v_{S'[abc]}$ are three phase ac voltages of the rectifier and of the inverter respectively; $i_{L2}$ is dc current of the DC cables; $R_1$, $L_1$, $R_3$ and $L_3$ are the equivalent resistors and reactors of the rectifier and of the inverter respectively; $R_2$ and $L_2$ are the equivalent resistors and reactors of DC cables; $C_1$ and $C_2$ are the DC capacitors of the rectifier and of the inverter respectively.

B. Time-domain Dynamic Model of VSC-HVDC

Because the three phase ac system is symmetric, taking phase a as the reference phase, Fig. 3 shows the equivalent circuit of phase a.

Fig. 3 Equivalent circuit of phase a of rectifier

For simplicity, $i(t)$ and $v(t)$ are shorted as $i$ and $v$, then:

$$L_1 \frac{di_{a1}}{dt} + R_1 i_{a1} = U_{Sa1} - v_{Fa} \quad (6)$$

Where $v_{Fa} = U_{C1} S_{a1} + v_{Hn}$, $S_{a1}$ and $S_{a1}'$ are the ideal switch-state functions of phase a, it is either 1 or 0 corresponding to on and off states of the switch respectively. Regardless of the adopted PWM scheme, $S_{a1}$ and $S_{a1}'$ are always complementary:

$$S_{a1} + S_{a1}' = 1 \quad (7)$$

For a balanced ac system, it is easy to derive:

$$v_{Hn} = -\frac{1}{3} U_{C1} \sum_{j=a,b,c} S_{j1} \quad (8)$$

Equations similar to (6) for phase ‘b’ and ‘c’ can also be developed. Substituting from (8) in (6), the final expression for three phases are deduced:

$$\begin{align*}
L_1 \frac{di_{a1}}{dt} & = -R_{ll1} - U_{C1} S_{a1} + \frac{1}{3} U_{C1} \sum_{j=a,b,c} S_{j1} + U_{Sa1} \\
L_1 \frac{di_{b1}}{dt} & = -R_{ll1} - U_{C1} S_{b1} + \frac{1}{3} U_{C1} \sum_{j=a,b,c} S_{j1} + U_{Sb1} \\
L_1 \frac{di_{c1}}{dt} & = -R_{ll1} - U_{C1} S_{c1} + \frac{1}{3} U_{C1} \sum_{j=a,b,c} S_{j1} + U_{Sc1}
\end{align*} \quad (9)$$

An equation similar to (9) can also be developed for inverter converter. Still considering the balanced operation of three phases, we can simplify the three phase model to reference phase a model only.

The switch state functions $S_{j1}$ and $S_{j3}$ are determined by the PWM control, and they are discrete, periodic function of time. For stability study, we only filter out the fundamental wave component and dc
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component of the switching functions $S_1$ and $S_3$, which can be expressed as:

$$
\begin{align*}
S_1 &= \frac{1}{2}m_1 \cos(\omega t - \delta_1 - \rho_1) + \frac{1}{2}m_0 \\
S_3 &= \frac{1}{2}m_2 \cos(\omega t - \delta_2 - \rho_2)
\end{align*}
$$

(10)

Here, $j = a, b, c$, $\rho_a = 0$, $\rho_b = 2\pi/3$, $\rho_c = 4\pi/3$. $m_1$ and $\delta_1$ are the modulation index and the trigger delayed angle of rectifier respectively, $m_2$ and $\delta_2$ are the modulation index and the trigger delayed angle of the inverter respectively.

Finally, the ac parts dynamics of VSC-HVDC can be described as:

$$
\begin{align*}
\frac{dI_{1a}}{dt} &= -R_{1a}I_{1a} - \frac{1}{2}m_1 \cos(\omega t - \delta_1) U_{C1} + U_{Sal} \\
\frac{dI_{1b}}{dt} &= -R_{1b}I_{1b} - \frac{1}{2}m_1 \cos(\omega t - \delta_1 - 2\pi/3) U_{C1} + U_{Sh1} \\
\frac{dI_{1c}}{dt} &= -R_{1c}I_{1c} - \frac{1}{2}m_1 \cos(\omega t - \delta_1 - 4\pi/3) U_{C1} + U_{Sc1} \\
\frac{dI_{3a}}{dt} &= -R_{3a}I_{3a} + \frac{1}{2}m_2 \cos(\omega t - \delta_2) U_{C2} + U_{Sa3} \\
\frac{dI_{3b}}{dt} &= -R_{3b}I_{3b} + \frac{1}{2}m_2 \cos(\omega t - \delta_2 - 2\pi/3) U_{C2} + U_{Sh3} \\
\frac{dI_{3c}}{dt} &= -R_{3c}I_{3c} + \frac{1}{2}m_2 \cos(\omega t - \delta_2 - 4\pi/3) U_{C2} + U_{Sc3}
\end{align*}
$$

(11)

From the Fig. 2, the dc parts dynamics of VSC-HVDC can be described as:

$$
\begin{align*}
C_1 \frac{dU_{C1}}{dt} &= i_{dc1} - i_{L2} = \sum_j j^* I_{ja} - j^* I_{ja} \\
L_2 \frac{dI_{L2}}{dt} &= U_{C1} - C_2 \frac{dU_{C2}}{dt} - i_{L2} R_{2a} \\
C_2 \frac{dU_{C2}}{dt} &= i_{L2} - i_{dc3} = i_{L2} - \sum_j j^* I_{ja} I_{ja} 
\end{align*}
$$

(12)

C. Dynamic Phasors Model of VSC-HVDC

The time-domain dynamic equations of VSC-HVDC are presented above. We assume that ac variables can be described accurately by their fundamental frequency components; and the dc variables can be described by the dc components. For the switching functions we consider the dc and fundamental frequency components. Based on these simplifications, the derivation of the corresponding dynamic phasors model of the above equation is given in this section.

Firstly, let’s consider phase a of the rectifier:

$$
\begin{align*}
\frac{d(i_{a1})}{dt} &= \frac{d(i_{a1})}{dt} + j\omega(i_{a1}) \\
\frac{d(i_{a1})}{dt} &= \frac{d(i_{a1})}{dt} - j\omega(i_{a1})
\end{align*}
$$

(13)

$i_{a1}(t)$ is a real signal, so

$$
(i_{a1})_{k} = (i_{a1})^* \text{, thus:}
$$

$$
\frac{dI_{1a}}{dt} = -j\omega I_{1a} - \frac{1}{2}m_1 U_{C1} + U_{Sal}
$$

(14)

Equations similar to (14) can also be developed for phase b and phase c:

$$
\frac{dI_{1b}}{dt} = -j\omega I_{1b} - \frac{1}{2}m_1 U_{C1} + U_{Sh1}
$$

(15)

$$
\frac{dI_{1c}}{dt} = -j\omega I_{1c} - \frac{1}{2}m_1 U_{C1} + U_{Sc1}
$$

(16)

We can deduce $I_{Lb1} = I_{La1} e^{-j\frac{2\pi}{3}}$, $I_{Lc1} = I_{La1} e^{-j\frac{4\pi}{3}}$, $I_{Sh1} = U_{Sa1} e^{-j\frac{2\pi}{3}}$, $I_{Lc1} = I_{La1} e^{-j\frac{4\pi}{3}}$ in the three phase balanced condition. Because the three phase ac system is symmetric, taking phase a as the reference phase.

We can get the corresponding dynamic phasors model of the rectifier capacitors.
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\[ \frac{d}{dt} \left( \langle d_{a1} \rangle \right) = \left( \sum_{j=a,b,c} i_{j} d_{j} \right) \]

Here:

\[ I_{La1} = \langle d_{a1} \rangle - 1 + \langle d_{a1} \rangle \]

Substituting above equations in (17), the final expression is deduced:

\[ \frac{dU_{C1}}{dt} = \frac{3m_{1} \cos \delta_{1}}{2C_{1}} I_{La1} - \frac{3m_{1} \sin \delta_{1}}{2C_{1}} L_{a1} - \frac{I_{L20}}{C_{1}} \]

Then, the corresponding dynamic phasors model of the inverter capacitors and dc cables are also deduced in the same way.

\[ \frac{dI_{20}}{dt} = \frac{U_{C10} - U_{C20}}{L_{2}} - \frac{R_{y} I_{L20}}{L_{2}} \]

\[ \frac{dU_{C20}}{dt} = \frac{U_{C10} - U_{C20}}{C_{2}} - \frac{3m_{2} \cos \delta_{2}}{2C_{2}} I_{La3} + \frac{3m_{2} \sin \delta_{2}}{2C_{2}} I_{La3} \]

Finally, the whole dynamic phasors model of VSC-HVDC can be gained by substituting the complex quantities of state variables into real parts and imaginary parts:

\[ X = AX + BU \]

Where

\[ X = \begin{bmatrix} i_{La1}^r & i_{La1}^i & U_{C10}^r & U_{C10}^i & U_{La3}^r & U_{La3}^i \\ \end{bmatrix}^T \]

\[ U = \begin{bmatrix} U_{S1}^r & U_{S1}^i & U_{S3}^r & U_{S3}^i \end{bmatrix}^T \]

\[ m_{2}, \delta_{2} \] are control variables, which can be acquired from the control system if VSC-HVDC.

D. The Control System of VSC-HVDC

The rectifier and inverter VSC control strategies adopted in this paper are shown in Fig. 4. \( K_{p} \) and \( K_{i} \) are the proportional gain and integral gain of the controller respectively. \( \delta_{0}, m_{0} \) are the steady state phase angle and modulation index of the VSC, \( \Delta \delta, \Delta m \) are the output of PI regulator [5].
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In general for a two-terminal VSC-HVDC system, one station must be set to use the constant DC voltage control.

4. SIMULATION RESULTS

The dynamic simulation of a VSC-HVDC transmission system is carried out in this section. Fig. 2 shows the circuit diagram of the VSC-HVDC transmission system. Detailed parameters are used given in Appendix A. These parameters are used to simulation both the dynamic phasors model and the EMT model using MATLAB/SIMLINK toolbox.

In order to verify the accuracy and efficiency of the dynamic phasors model, we compare the simulation results of the dynamic phasors model with those of EMT model under two typical power controlled mode based on the system shown in Fig. 2.

Case 1: At 1s, the reference value of the inverter active power steps from 3MW to -3MW, at 3s, it steps from -3MW to 3MW. The simulation results of using the dynamic phasors model and the EMT model are given in the Fig. 5(a) and Fig. 5(b) respectively.

Case 2: At 1s, the reference value of the inverter reactive power steps from 0Mvar to -2Mvar, at 3s, it steps from -2Mvar to 0Mvar. The simulation results of using the dynamic phasors model and the EMT model are given in the Fig. 6(a) and Fig. 6(b) respectively.

As seen from the simulation results, dynamic phasors model can reflect the dynamic behavior correctly. The results of the VSC-HVDC system using EMT model and dynamic phasors model are consistent. The test illustrates that simulation results using dynamic phasors model have satisfied accuracy as compared with EMT model.
Furthermore, we compare the computation time as shown in Table I of the two models, and find that the time consumed by using the dynamic phasors model is far less than that of the EMT models. Simulations are performed using an Intel CPU with 1.13GHz and 512MB RAM memory under the MATLAB version 6.5.0.180391a(Release13) environment.

**TABLE I:** Comparison of the simulation time of the two models

<table>
<thead>
<tr>
<th>VSC-HVDC model</th>
<th>Simulation interval(s)</th>
<th>Computation time(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic phasors</td>
<td>0 ~ 5</td>
<td>6.1480</td>
</tr>
<tr>
<td>EMT</td>
<td>0 ~ 5</td>
<td>393.0233</td>
</tr>
</tbody>
</table>

5. CONCLUSION
A new modeling method has been introduced in this paper and simulation VSC-HVDC transmission system. Through the simulation results, we can conclude that:

a) The dynamic phasors method can model VSC-HVDC dynamic accurately and efficiently;
b) The dynamic phasors is more accurate than quasi-static model and simpler than detailed EMT model in analysis and simulation;
c) Dynamic phasors is flexible to include more harmonics for a better accuracy (at a price of increased model complexity) when needed;
d) Comparison with the detailed time-domain model, dynamic phasors model can save much CPU time;
e) The dynamic phasors model of VSC-HVDC is modular, and completely compatible with other models in the whole system.

6. APPENDIX
The parameters of the VSC-HVDC transmission systems used in this paper is as follows.

\[ U_{S1}=\sqrt{2}\ kV, \ U_{S3}=\sqrt{2}\ kV, \ f=50\ Hz, \]
\[ R_1=R_3=0.20\Omega, \ R_2=1.0\Omega, \ C_1=C_2=7\ \text{mF}, \]
\[ L_1=L_3=7\ \text{mH}, \ L_2=7\ \text{mH}, \ N=\frac{f_r}{f}=21. \]

Control system: a constant voltage...
controller and a constant reactive power
controller are used in rectifier station; a
constant active power controller and a
constant reactive power controller are used
in inverter station.

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Biographies

Wei Yao was born in Hubei, China in 1983. He received his B.S. degrees from Huazhong University of science and technology (HUST), China in 2004, where he is currently pursuing the Ph.D. degree. His research interests are power system control and modeling.

Jinyu Wen received the B.S. and Ph.D. degrees in electrical engineering from Huazhong University of Science and Technology (HUST), Wuhan, China, in 1992 and 1998, respectively. He is a Professor at HUST. He was a Postdoctoral Researcher with HUST from 1998 to 2000, and the Director of Electrical Grid Control Division, XJ Relay Research Institute, Xuchang, China, from 2000 to 2002. His research interests include evolutionary computation, intelligent control, power system automation, power electronics and energy storage.

Shijie Cheng graduated from the Xi’an Jiaotong University, Xi’an, China in 1967 and received a Master of Engineering Degree from the HUST, Wuhan, China in 1981 and a Ph.D. from the University of Calgary, Calgary, Canada in 1986 all in the Electrical Engineering. He is now a full professor at the HUST. His research interests are power system control, stability analysis of power system and application of AI in power systems.