ADAPTIVE FUZZY SLIDING MODE CONTROL FOR A CLASS OF BIPARTITE MODULAR ROBOTIC SYSTEMS

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ABSTRACT

One of the fundamental issues in the field of modular robotics is the design and implementation of robotic systems with low cost and high performance. Although the coordination of modules entails the minimization of several different performance criteria, the success of the evolution of a group of the modules is strictly dependent upon the fulfillment of control goals. The cost of the modules, on the other hand, is subject to the components used to measure the state, and the hardware used for actuation. Therefore, obtaining a good control performance with cheap hardware is a challenge for control specific issues. In this paper, we describe an adaptive fuzzy sliding mode control scheme implemented on the control of 3-DOF I-Cubes links, which operate in a highly information-limited environment due to the size constraints, and which are bipartite. The tuning law is justified both in continuous time and in discrete time cases of sliding mode control approach. The implementation results justify the theoretical foundations and strongly recommend the approach due to its low computational cost together with the robustness against disturbances and uncertainties.

Keywords: Tuning Laws, Fuzzy Control, Robotics, and Sliding Mode Control

I. INTRODUCTION

The idea of modular robotic systems has been inspired from the biological counterparts, which can adapt themselves to form topologically different objects with a single and massively interconnected entity. Several examples can be visualized in the cell level biology of microorganisms. However, at a larger scale with man made machines, the concept of coordination and cooperation of individual cells depends heavily on the control of the interactions between...
the modules. At this stage, the low level control performance enters into the picture and plays a central role in achieving the precise realization of the task. One challenge that constrains the design is the cost of the design. Since the design is subject to the individual control of a number of modules, and since the ultimate goal of the design is to obtain autonomous reconfiguration with low cost, it becomes inevitable to come up with robust control schemes that handle uncertainties arising in a highly information-limited environment. More explicitly, the controller for each actuator must be capable of alleviating the drawbacks brought about by considerably low sampling rate, noise, low encoder resolution, unknown and nonlinear friction issues in gears and dead zone issues. Obviously, in such an environment, the adopted controller structure must be simple enough in order not to violate the computational requirements and must be versatile enough to fulfill the desired task.

Various applications analyzing the capabilities of self-reconfigurable or modular robotic systems are reported in the literature. Early studies on this topic report manual configuration [1], a modularly synthesized kinematics structure called Tetrabot [2], and cellular mobile robots with reconfiguration capability [3]. Later studies have reported examples in 2D such as metamorphing hexagonal modules [4], self-repairing machines [5], the Cristalline robot [6], which moves in a horizontal plane, and Inchworm [7], which moves in a vertical plane. The examples operating in 3D are Polypod/Polybot [8], CONRO [9], robotic molecule [10], self-reconfigurable structure [11], modular robot [12], Proteo [13] and I-Cubes [14]. A common property of what is presented in [8-14] is the capability of exploiting neighboring modules to fulfill the task.

When the modularity is achieved through mechanical actuation based on motors, the issues related to the control engineering expertise have to be studied carefully. The designed hardware and the adopted control strategy must be in good compliance to achieve the overall tracking accuracy, which is a prerequisite for the success of self-reconfiguration trials. Since the nature of the problem is involved with the alleviation of a number of difficulties, which are frequently encountered in practice, the solution should be sought in the domain of intelligent control systems. In the literature, many architectures of intelligent control schemes have been discussed. Artificial neural networks, fuzzy inference systems and those utilizing genetic algorithms or the hybrid variants of these are just to name a few [15]. In this paper, we utilize an adaptive Fuzzy Logic Controller (FLC) processing the error and the estimated rate of error in discrete time and producing a control signal to drive the module under control towards a predefined regime.

In physical implementations of intelligent controllers, the issues of parameter tuning must be contemplated with particular care. The reason for this is the fact that the parameters of the controller are to be refined at each sampling instant but the update mechanism will need a critic information containing the amount of discrepancy on the applied control signal. Apparently, the nature of the control problems does not allow the existence of such kind of supervision; rather, the designer is forced to extract the control error from the observed quantities. One important result presented in this paper addresses this problem from an analytic point of view. Furthermore, it is shown that the system under control is driven towards a predefined sliding regime by the suggested tuning law and error measure.

Sliding Mode Control (SMC) is a well-known nonlinear control approach, which is based on a two-sided decision mechanism. The approach introduces certain degrees of robustness against disturbances and uncertainties due to the stable and attracting invariant loci created in the phase space. The underlying idea is to force the tracking error vector towards the attracting loci, which is the switching hyperplane, and to maintain the motion on this multidimensional surface. The control strategy has extensively been studied for tracking control of uncertain systems, whose nominal governing dynamics are known with the bounds of uncertainties. A thorough investigation of the concept is presented in [17-19], and the relevance of the technique with intelligent control schemes is figured out in [20] from the point of how SMC could be used for intelligent control and how intelligence could be used to improve the performance of SMC schemes.
Although the SMC concept has been exemplified in various applications, when the discrete time implementations are considered, the results integrating the issues of intelligence and control in the context of a rigorous stability analysis are still in their way towards maturity. Several works focusing on the Discrete Time Sliding Mode Control (DTSMC) approach report the necessary and sufficient conditions for stability [21], use of reaching law approach in discrete time [22], convergence issues in quasi-sliding mode [23], and selection of the sampling time [24]. One of the examples demonstrating the feasibility and efficacy of such approaches in the field of DTSMC focuses on an exhaust measuring system [25], which introduces the use of a fuzzy supervisor utilizing triangular membership functions. The work presented by Xu et al [26] adopts a hybrid approach based on neuro-fuzzy integration, by the use of which the DTSMC task is achieved by tuning the parameters of a neuro-fuzzy identifier. The tuning is performed through error backpropagation technique, and the results have been discussed for computer control of a two link robotic manipulator. In [27] and [28], the design of DTSMC with recurrent neural networks and Gaussian radial basis function neural networks is presented respectively. In both studies, the neural networks are utilized for estimation purposes.

In the next section, we introduce the I-Cubes system and the control loop, next the fuzzy controller and its adaptation scheme is presented. In the fourth section, we present the implementation results, and the concluding remarks are given at the end of the paper.

2. I-CUBES

In this paper, we consider a class of self-reconfigurable modular robots to demonstrate the control performance of the proposed approach. Previous research in I-Cubes project have demonstrated that the concept of self-reconfiguration could be attained autonomously through a self decomposition of the task into subtasks and executing a planning strategy based on the extracted decisions in software domain [14].

I-Cubes system discussed in [14,29-31] is a bipartite robotic system composed of cubes forming a lattice and links (3-DOF manipulators) providing connections between the lattice elements. The components and the different CAD views of an I-Cubes link module are shown in Fig.1. In Fig. 1(a) the motor used for each joint is shown. The motor is Cirrus CS 21-BB Hi Performance Sub-Micro BB Servo. Its specifications are as follows: Size (cm) 2.1844×1.9812×1.0922; Weight (gr) 9.0651; Speed (sec/60º) 0.09 under 4.8V, 0.078 under 6.0V; Torque (gr-cm) 1367.1387 under 4.8 V, 1654.9573 under 6.0 V. Fig. 1 (b) demonstrates how the motor is installed at the end of a single link. In Fig. 1 (c) and (d), the gearing that transfers rotary the motion of the motor to the link end is shown. A compact view is illustrated in Fig. 1(e) with encoders and two motors at the link ends and one motor in the middle of the whole system. Since the used encoders are 24-CPR and the worm gear reduction is 1:30, the resolution of the encoder feedback is 360º/(30×24)=0.5º. The cross-shaped connectors at the link ends are designed for attachment and detachment and are not under investigation in this paper. The last subplot of Fig. 1 demonstrates a single module of I-Cubes system used in the tests. As depicted in Fig. 1(e) and (f), the system has two links, each having six centimeters length, and three actuators to provide mobility in 3D space.

The block structure of the controlled system is depicted in Fig. 2, in which each motor is operating under an ordinary feedback loop. At each control instant, PIC sends 3 bytes to the computer. Two bytes of this information hold the measured position and the remaining byte is for the speed in terms of pulses and pulses/sec respectively. On the other hand, the computer sends two bytes to the PIC. First byte is for the direction (or sign) and the other is for PWM (Pulse Width Modulation) duty-cycle. The described process is achieved through a serial communication between the computer and the PIC. The two systems are synchronized at a rate of 160 msec (Tc), which is the adopted value of the control period. The process that takes place inside the computer include the interfacing, that has been designed on Java 2 platform, the control algorithm and parameter tuning scheme yet to be discussed, and the serial communication software. The first apparent challenge here is the width of control period.
The system will be forced to find the desired state through a set of observations that are sparse in time. One might suggest increasing the frequency, but the practical difficulty is that the optical encoder does not provide sufficiently accurate readings when the sampling frequency is increased, and the observed positional data decreases to the order of a few pulses, which lead to an excessive noise when the speed is evaluated. On the other hand, changing the encoder and the relevant components with those providing better accuracy conflicts with the cost and size objectives of the I-Cubes system. In what follows, we describe the controller and the parameter tuning mechanism.

3. FUZZY LOGIC CONTROLLER AND THE ADAPTATION MECHANISM

3.1. The Controller
Contrary to what is postulated in the realm of predicate logic, representation of knowledge by fuzzy quantities can provide extensive degrees of freedom if the task to be achieved can better be expressed in words than in numbers. The concept of fuzzy logic in this sense can be viewed as a generalization of binary logic and refers to the manipulation of knowledge with sets, whose boundaries are unsharp [32]. Therefore the paradigm offers a possibility of designing intelligent controllers operating in an environment, in which the conditions are inextricably intertwined and subject to uncertainties and impreciseness.

Understanding the information content of fuzzy logic systems is based on the subjective judgements, intuitions and the experience of an expert. From this point of view, a suitable way of expressing the expert knowledge is the use of IF antecedent THEN consequent rules, which can easily evaluate the necessary action to be executed for the current state of the system under investigation.
Structurally, a FLC is comprised of five building blocks, namely, fuzzification, inference engine, knowledge base, rule base, and defuzzification. Since the philosophy of the fuzzy systems is based on the representation of knowledge in a fuzzy domain, the variables of interest are graded first. This grading is performed through the evaluation of membership values of each input variable in terms of several class definitions. According to the definition of a membership function, how the degree of confidence changes over the domain of interest is characterized. This grading procedure is called fuzzification. In the knowledge base, the parameters of membership functions are stored. Rule base contains the cases likely to happen, and the corresponding actions for those cases through linguistic descriptions, i.e. the IF-THEN statements. The inference engine emulates the expert’s decision making in interpreting and applying knowledge about how the best fulfillment of the task is achieved. Finally, the defuzzifier converts the fuzzy decisions back onto the crisp domain [23].

The architecture utilized in this paper uses algebraic product operator for the aggregation of the rule premises and triangular membership functions denoted by \( \mu \). The overall representation of this structure is given in (1), in which \( R \) and \( m \) stand for the number of rules contained in the rule base and the number of inputs of the structure. The output of the FLC is evaluated by using weighted average defuzzification scheme. Throughout the paper, an underlined variable should be understood as a vector of appropriate dimensions.

\[
\Omega = \phi^T \Omega (1)
\]

with the \( i^{th} \) rule as: IF \( v_1 \) is \( \exists_{i1} \) AND \( v_2 \) is \( \exists_{i2} \) AND ... AND \( v_m \) is \( \exists_{im} \) THEN \( f_i = \phi_i \). In the IF part of this representation, the lowercase variables denote the inputs and \( \exists_{ij} \)s stand for the fuzzy sets corresponding to the domain of each linguistic label. The THEN part is comprised of the prescribed decision, which is denoted by \( f_i \), in the form of a scalar number denoted by \( \phi_i \). Generically, the adjustable parameters of the structure are comprised of the parameters of the membership functions together with the defuzzifier parameters \( \phi_i \). Since \( \Omega \) is the vector of fuzzy basis functions, an important feature of the representation in (1) is the linearity of the output in the defuzzifier parameters. In Fig. 3, (to which more reference will be made at a later stage, explaining the notations used on the figure) how the input space is covered by the membership functions is depicted.
In this paper, each one of the motors in a single module is controlled by a separate FLC of structure given in (1). Since the controllers for all three motors have the same structure, we skip subsystem indexing in order to avoid confusion. The inputs to a FLC are the tracking error and its rate, i.e. $v_1 = e$ and $v_2 = \dot{e}$ measured at discrete instants of time. The inputs are in encoder pulses for errors and pulses per second for rates of errors, whereas the output of each FLC is a crisp integer number determining the duty-cycle of the Pulse Width Modulation (PWM) pulses to be produced by the PIC 16C63A shown in Fig. 2.

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3.2. Tuning Mechanism for the Motors of an I-Cubes Module

Consider the system given as
\[ \dot{x}(t) = \Psi(x(t), \dot{x}(t)) + \Phi(x(t))u(t), \]
where
\[ x = (\theta_1, \theta_2, \theta_3)^T \]
denote the angular position of each motor, and
\[ u = (u_1, u_2, u_3)^T \]
is the vector of inputs for the three motors. Defining
\[ \omega = (\omega_1, \omega_2, \omega_3)^T \]
as the angular velocities, and assuming the coupling effects as functions of time, the governing dynamics of each one of the actuators in the module resembles to the one given in (2).
\[ \dot{\theta}(t) = \omega(t) \] and
\[ \dot{\omega}(t) = P(\theta(t), \omega(t), t) + Q(t)u(t) \] (2)

Let us drop the time variable and denote the desired angular positional behavior by \( \theta_d \), which is the command signal. The goal of the design is to force the error dynamics towards the sliding manifold described by \( s_p \), where
\[ s_p = \dot{e} + \lambda e, e = \theta_d - \theta \]
and \( \lambda \) is the slope of the sliding manifold. Obviously if the functions \( P \) and \( Q \) were known exactly, or the bounds of the uncertainties on these functions were known, one would suggest the sequence given as
\[ u_{smc} = Q^{-1}(\omega_d + \lambda e + \alpha \text{sgn}(s_p) + \beta s_p - P), \]
with \( \lambda > 0, \alpha > 0 \) and \( \beta > 0 \). The formulated sequence enforces \( \dot{s}_p = -\alpha \text{sgn}(s_p) - \beta s_p \), which corresponds to constant plus proportional rate reaching law of SMC terminology [17]. It should be clear that the application of \( u_{smc} \) would enforce the error vector to hit the sliding manifold. The behavior during this phase is characterized by \( \dot{s}_p = -\alpha \text{sgn}(s_p) - \beta s_p \). Once the error vector gets trapped to the subspace characterized by \( s_p = 0 \), the behavior imposed along the locus \( s_p = 0 \) would bring the error vector towards the origin of the phase space.

**Definition 1.** The error on the control signal is defined to be the discrepancy between \( u_{smc} \) and the produced control \( u \). This quantity can explicitly be defined as \( s_e = u_{smc} - u \).

**Proposition 2.** The error caused by the controller, i.e. \( s_e = u_{smc} - u \), can be estimated as \( \dot{s}_e = \dot{s}_p + \alpha \text{sgn}(s_p) + \beta s_p \), the minimization of which in magnitude ensures reaching and convergence to origin due to the dynamics of the sliding manifold.

The above proposition suggests that the tuning of the controller parameters are performed only if the error vector is not on the sliding manifold. Consider the two surfaces shown in Fig. 4. Most of the tuning schemes define the origin as the point at which the cost is minimal (See Fig 4(a)). However, the selection in the above proposition augments the set of points that minimize the cost as illustrated in Fig 4(b), hence, the convergence does not take a long time as in the case of traditional tuning schemes.

**Remark 3.** In this study, there are three FLCs, i.e. one for each motor, and the vector of the defuzzifier parameters (\( \Phi \)) of them are adjusted individually. In order not to be in conflict with the physical reality, for each FLC, it is assumed that the adjustable parameters, the time derivative of the signal exciting the controller and the time derivative of the target output of the controller remain bounded, i.e.
\[ \|\| \leq B_\|, \]
\[ \|\| \leq B_\| \text{ and } \|u_{smc}\| \leq B_{u_{smc}}. \]
Theorem 4. For a system of structure (2) and a FLC of structure (1), the adaptation of controller parameters as described in (3) enforces the value of the control discrepancy \( \sigma_c \) to zero.

\[
\dot{\theta}_{k+1} = \theta_k + T_s \omega_k \quad \text{and} \quad \omega_{k+1} = \omega_k + T_s \left( p \left( \theta_k, \omega_k, k \right) + \Omega(k \mu(k)) \right)
\]

(6)

where, \( \gamma \) is a sufficiently large positive constant satisfying

\[
\gamma > B_p B_\Omega + B_{\text{smc}}
\]

(4)

The adaptation mechanism in (3) drives an arbitrary initial value of \( \sigma_c \) to zero in finite time denoted by \( t_h \) satisfying the inequality in (5).

\[
t_h \leq \frac{\| \sigma_c(0) \|}{\gamma - (B_p B_\Omega + B_{\text{smc}})}
\]

(5)

Proof: The proof of a similar case is presented in [34], which assumes the availability of the target outputs.

Theorem 5: If the system enters the sliding mode \( \sigma_c = 0 \) and remains in it thereafter, then the parameters of the FLC, \( \phi \), evolve bounded.

Proof: Refer to [35].

3.3. Analysis of the Scheme from the DTSMC Perspective

The update law of (3) can be discretized by using first order Euler approximation and the stability issues can formally be analyzed as given next. The discrete time index is shown as a subscript with integer variable \( k \).

Assumption 6. There exists a number \( \Gamma \) such that \( \frac{(\Omega_k^T \Omega_{k+1})}{(\Omega_k^T \Omega_k)} > \Gamma > 0 \) is satisfied in \( \Re^m \), where \( m \) is the number of FLC inputs and is equal to 2. The meaning of this and the conditions under which the above inequality holds true will be discussed later.

Theorem 7. For a discrete time system of structure (6), the use of a two input one output FLC described in (1) with a parameter adaptation rule as described in (7) leads to \( s_{ck} \{ s_{ck+1} - s_{ck} \} < 0 \), and the system is driven towards the quasi-sliding regime characterized by (8).

\[
\begin{align*}
\dot{\theta}_{k+1} &= \theta_k + T_s \omega_k \quad \text{and} \quad \\
\dot{\omega}_{k+1} &= \omega_k + T_s \left( p \left( \theta_k, \omega_k, k \right) + \Omega(k \mu(k)) \right) \\
\phi_{k+1} &= \dot{\phi}_k + T_s \frac{\Omega}{\Omega_k} \gamma \text{sgn}(s_{ck}) \\
s_{pk+1} &= s_{pk} - T_s \left( \alpha \text{sgn}(s_{pk}) + \beta s_{pk} \right)
\end{align*}
\]

(7)

(8)

where \( T_s \) is the sampling rate and \( \gamma \) is constrained to satisfy \( \gamma > \frac{\zeta}{\Gamma} \) with \( \zeta = 2(B_p B_\Omega + B_{\text{smc}}) \).

Proof: Define \( \Omega_{k+1} = \Omega_k + \Delta \Omega_{k+1} \) and \( u_{\text{smc},k+1} = u_{\text{smc},k} + \Delta u_{\text{smc},k+1} \). It should be clear that \( \| \Delta \Omega_k \| \leq 2B_\Omega \) and \( \| \Delta u_{\text{smc},k} \| \leq 2B_{\text{smc}} \) for \( \forall k \geq 0 \), where \( B_\Omega \) and \( B_{\text{smc}} \) are some positive constants satisfying \( \| \Omega_k \| \leq B_\Omega \) and \( \| u_{\text{smc},k} \| \leq B_{\text{smc}} \) for \( \forall k \geq 0 \) respectively. Furthermore, the parameters of the FLC are assumed to be bounded, i.e. \( \| \phi_k \| < B_p \) with \( B_p > 0 \). Using these quantities, and referring to the definition and estimation of \( s_{ck} \) in Proposition 2 with (1), one can proceed as follows:

As long as the inequality in Assumption 6 holds true, the obtained result implies that the adaptation mechanism enforces the FLC to synthesize \( u_{\text{smc}} \) which forces \( s_{ck} \to 0 \) or equivalently the reaching law of (8), and leads to the achievement of the prescribed DTSMC task, and the theorem is proved.
that can be observed from $A_1 \rightarrow A_6$ transition is strictly positive. For this reason, consider the data given in Table 1, in which we summarize the results for all possible transitions. One should notice that there are equivalent transitions leading to the same least value. These are given in the third column of the table. In analyzing the results we enumerate the rules as follows: {Rule#1:NN, Rule#2:NZ, Rule#3:NP, Rule#4:ZN, Rule#5:ZZ, Rule#6:ZP, Rule#7:PN, Rule#8:ZP, Rule#9:PP}. Among what is given in the fourth column of Table 1, the transitions $A_6 \rightarrow A_{10}$, $A_7 \rightarrow A_{10}$, $A_9 \rightarrow A_{11}$, and $A_{12} \rightarrow A_{11}$ are the most difficult ones as they require the evaluation of four rule outputs. For example in $A_6 \rightarrow A_{10}$ transition we get

$$ s_{ik}(s_{ik+1} - s_{ik}) = s_{ik}(u_{A_{11},2} - \varphi^T T \Omega_k + L_i) $$

where, $0<\Lambda_1<1$ and $0<\Lambda_2<1$.

Proof: Since $\gamma T > \zeta / \Gamma$ and we assumed that $\left( \frac{\Omega_k^T \Omega_{k+1}}{\Omega_k^T \Omega_k} \right) / \Gamma > 0$ (Refer to Assumption 6), what we need is to evaluate the least value of $\left( \frac{\Omega_k^T \Omega_{k+1}}{\Omega_k^T \Omega_k} \right)$ and to show that it is strictly positive. Before going into the details, one should notice from (10)-(11) that a binary change in any of the membership functions is prohibited. For example, if $\mu_{i_1}(v_{1,k}) = 1$ for some $k$, the value of $\mu_{i_2}(v_{1,k+1})$ can decrease at most to the level $1-\Lambda_1$. Referring to Fig. 3, let the input vector perform a transition from region $A_1$ at time $k$ to region $A_{10}$ at time $k+1$, and denote this transition by $A_1 \rightarrow A_6$. Clearly, the conditions in (10)-(11) require that the point $v_2$ in Fig. 3 can reach points in the shaded area at time $k+1$, and this area is the largest area that can be reached from the region $A_1$ due to the conditions in (10)-(11). Having this in mind, we can claim that the least value of $\Omega_k^T \Omega_{k+1}$ that can be observed from $A_1 \rightarrow A_6$ transition is $A_1 A_2$. In obtaining this, one should note that it is sufficient to check the least value of $\Omega_k^T \Omega_{k+1}$ because the supremum value of $\Omega_k^T \Omega_{k+1}$ is unity. Once the minimal least value of $\Omega_k^T \Omega_{k+1}$ for all possible transitions is constructed, existence of a strictly positive $\Gamma$ value can be claimed if the globally minimum value of $\Omega_k^T \Omega_{k+1}$ is strictly positive. For this reason, consider the data given in Table 1, in which we summarize the results for all possible transitions. One should notice that there are equivalent transitions leading to the same least value. These are given in the third column of the table. In analyzing the results we enumerate the rules as follows: {Rule#1:NN, Rule#2:NZ, Rule#3:NP, Rule#4:ZN, Rule#5:ZZ, Rule#6:ZP, Rule#7:PN, Rule#8:ZP, Rule#9:PP}. Among what is given in the fourth column of Table 1, the transitions $A_6 \rightarrow A_{10}$, $A_7 \rightarrow A_{10}$, $A_9 \rightarrow A_{11}$, and $A_{12} \rightarrow A_{11}$ are the most difficult ones as they require the evaluation of four rule outputs. For example in $A_6 \rightarrow A_{10}$ transition we get

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where, $0<\Lambda_1<1$ and $0<\Lambda_2<1$.
which is to be evaluated over a domain satisfying the following conditions:

i) \( v_{1,k} \), \( v_{1,k+1} \in [-L_1, 0] \)

ii) \( v_{2,k}, v_{2,k+1} \in [0, L_2] \)

iii) \( \| v_{1,k} - v_{1,k+1} \| \leq (1 - \Lambda_1) L_1 \)

iv) \( \| v_{2,k} - v_{2,k+1} \| \leq (1 - \Lambda_2) L_2 \)

In Fig. 5, we portray the hexagonal domains over which the inframum values are to be evaluated. More explicitly, \( \inf_{v_{1,k}, v_{1,k+1} \in A_6} F(v_{1,k}, v_{1,k+1}) \) is evaluated over the hexagon \( \sigma_1 \sigma_2 \sigma_3 \sigma_4 \sigma_5 \) and \( \inf_{v_{2,k}, v_{2,k+1} \in A_6} G(v_{2,k}, v_{2,k+1}) \) is evaluated over \( \sigma_6 \sigma_7 \sigma_8 \sigma_9 \sigma_{10} \). The function \( F \) takes it minimal values at the centers of the line segments \( \sigma_1 - \sigma_2 \) and \( \sigma_4 - \sigma_5 \), and the function \( G \) takes its minimal values at the centers of the line segments \( \sigma_6 - \sigma_7 \) and \( \sigma_9 - \sigma_{10} \). When the corresponding values are solved, the least value of \( \Omega_{k+1} \) is obtained as given in the \( A_6 \) row of Table 1. In finding the expressions in the fourth column of Table 1, one should keep in mind that \( \sum_{i=1}^{R} \prod_{j=1}^{m} \mu_i (v_j) = 1 \) due to the selection of membership functions depicted in Fig. 3. In accordance with this, the entries of \( \Omega_k \) vector contain the multiplication of relevant membership grades for the two inputs.

If the fourth column of the table is combined, we get the final result given below:

\[
\frac{\Omega_{k+1}^T \Omega_k}{\Omega_k} > \Lambda_1 \Lambda_2 \min \left( \left| 1 - \frac{\Lambda_1}{2} \right|, \left| 1 - \frac{\Lambda_2}{2} \right|, \Lambda_1 \Lambda_2 \right) \tag{13}
\]

The result above ensures that a \( \Gamma > 0 \) exists in \( \Re^2 \) (or in \( v_{1,k} \times v_{2,k} \) for \( k \geq 0 \)) and it satisfies the inequality in (14).

\[
0 < \Gamma < \Lambda_1 \Lambda_2 \min \left( \left| 1 - \frac{\Lambda_1}{2} \right|, \left| 1 - \frac{\Lambda_2}{2} \right|, \Lambda_1 \Lambda_2 \right) \tag{14}
\]

The result given above proves Theorem 8 and confirms the convergence claim of Theorem 7. □

Remark 9. A system of structure (2) in continuous time, or equivalently of structure (6) in discrete time with a FLC of input-output relation (1) with membership functions set as depicted in Fig. 3 can be driven towards the predefined sliding regime by tuning the defuzzifier parameters through the adoption of (3) for continuous time realizations, or (7) for sampled data systems.

Considering the inequality in (4) and the last line of (9), one can explicitly suggest the following relation for uncertainty bound parameter.

\[
\gamma > \max \left( B_{\text{b}} B_{\Omega}, B_{\text{b}_{\text{unc}}}, \frac{\zeta}{\Omega_T} \right) \tag{15}
\]

The relation above merges the results of continuous time case and discrete time case.

4. EXPERIMENTATION AND OBSERVATIONS

In this section, we discuss the results obtained through real-time experimentation of the I-Cubes module. Several difficulties related to the experimentation are emphasized, and the way in which the high performance with low cost is observed is clarified.

The first issue in the implementation stage is the time sparseness of the observed data and the necessity of filtering. Denote the Laplace variable by \( S \) and discretize the filter structure \( H(S) = H(S+) \) by forward Euler approximation. During the tests, one major problem alleviated by such a simple filtering is the noise contained in the calculated angular velocities, and the evaluated FLC outputs.
Table 1. Possible Regional Transitions and the Least Value of $\Omega_k^T \Omega_{k+1}$

<table>
<thead>
<tr>
<th>Regional Transition</th>
<th>Contributing Rule(s)</th>
<th>Equivalent Transitions</th>
<th>Least value of $\Omega_k^T \Omega_{k+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1 \leftrightarrow A_1$</td>
<td>3</td>
<td>$A_4 \rightarrow A_4$, $A_{13} \rightarrow A_{13}$, $A_{16} \rightarrow A_{16}$</td>
<td>$\lambda_1$</td>
</tr>
<tr>
<td>$A_1 \leftrightarrow A_2$</td>
<td>3</td>
<td>$A_4 \rightarrow A_4$, $A_{13} \leftrightarrow A_{13}$, $A_{16} \leftrightarrow A_{16}$</td>
<td>$\lambda_2$</td>
</tr>
<tr>
<td>$A_1 \leftrightarrow A_3$</td>
<td>3</td>
<td>$A_4 \rightarrow A_4$, $A_{13} \leftrightarrow A_{13}$, $A_{16} \leftrightarrow A_{16}$</td>
<td>$\lambda_1 \lambda_2$</td>
</tr>
<tr>
<td>$A_2 \rightarrow A_2$</td>
<td>3, 6</td>
<td>$A_3 \rightarrow A_3$, $A_{14} \rightarrow A_{14}$, $A_{15} \rightarrow A_{15}$</td>
<td>1</td>
</tr>
<tr>
<td>$A_2 \leftrightarrow A_3$</td>
<td>6</td>
<td>$A_{14} \leftrightarrow A_{15}$</td>
<td>$\lambda_1^2$</td>
</tr>
<tr>
<td>$A_2 \leftrightarrow A_4$</td>
<td>3</td>
<td>$A_4 \leftrightarrow A_4$, $A_{13} \leftrightarrow A_{13}$, $A_{16} \leftrightarrow A_{16}$</td>
<td>$\lambda_1 \lambda_2$</td>
</tr>
<tr>
<td>$A_2 \leftrightarrow A_5$</td>
<td>3, 6</td>
<td>$A_4 \leftrightarrow A_4$, $A_{13} \leftrightarrow A_{13}$, $A_{16} \leftrightarrow A_{16}$</td>
<td>$2\lambda_1 \lambda_2$</td>
</tr>
<tr>
<td>$A_3 \rightarrow A_3$</td>
<td>2, 3</td>
<td>$A_3 \rightarrow A_3$, $A_{13} \rightarrow A_{13}$, $A_{16} \rightarrow A_{16}$</td>
<td>1</td>
</tr>
<tr>
<td>$A_4 \leftrightarrow A_4$</td>
<td>2, 3</td>
<td>$A_4 \leftrightarrow A_4$, $A_{13} \rightarrow A_{13}$, $A_{16} \rightarrow A_{16}$</td>
<td>$2\lambda_1 \lambda_2^2$</td>
</tr>
<tr>
<td>$A_4 \rightarrow A_4$</td>
<td>2</td>
<td>$A_4 \leftrightarrow A_4$, $A_{13} \rightarrow A_{13}$, $A_{16} \rightarrow A_{16}$</td>
<td>$\lambda_2^2$</td>
</tr>
<tr>
<td>$A_5 \rightarrow A_5$</td>
<td>2, 5</td>
<td>$A_5 \leftrightarrow A_5$, $A_{13} \rightarrow A_{13}$, $A_{16} \rightarrow A_{16}$</td>
<td>$2\lambda_1^2 \lambda_2^2$</td>
</tr>
<tr>
<td>$A_6 \rightarrow A_6$</td>
<td>5, 6</td>
<td>$A_6 \leftrightarrow A_6$, $A_{10} \rightarrow A_{10}$, $A_{11} \rightarrow A_{11}$</td>
<td>$\lambda_1 \lambda_2 (1 - 0.5 \lambda_1)(1 - 0.5 \lambda_2)$</td>
</tr>
<tr>
<td>$A_8 \rightarrow A_8$</td>
<td>2</td>
<td>$A_8 \leftrightarrow A_8$, $A_{12} \rightarrow A_{12}$</td>
<td>$2\lambda_1 \lambda_2^2$</td>
</tr>
<tr>
<td>$A_{10} \rightarrow A_{10}$</td>
<td>2</td>
<td>$A_{10} \leftrightarrow A_{10}$, $A_{12} \rightarrow A_{12}$</td>
<td>$\lambda_1^2 \lambda_2$</td>
</tr>
</tbody>
</table>

F if $j=1$ or G if $j=2$

*Fig. 5 Regions of Interest for $A_6 \rightarrow A_6$ Transition or its Equivalents*
Fig. 6 Tracking errors and applied control signals

Table 2. Real-Time Experimentation Parameters for All Three FLCs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>5.9375</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>4</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\tau_u$</td>
<td>3.125</td>
</tr>
<tr>
<td>$\tau_v$</td>
<td>3.125</td>
</tr>
<tr>
<td>$L_1$</td>
<td>100 ($5\pi/36$ rad)</td>
</tr>
<tr>
<td>$L_2$</td>
<td>20 ($\pi/36$ rad/sec)</td>
</tr>
<tr>
<td>$U_{max}$</td>
<td>6</td>
</tr>
<tr>
<td>$U_{min}$</td>
<td>-6</td>
</tr>
</tbody>
</table>

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For this purpose, we filter the velocity error before applying it to the FLCs and the output voltages before driving the motors. The filter structures for the velocity error and the control input are described in (16) and (17) respectively.

\[
\dot{\hat{v}}_{2,k+1} = (1 - \tau_v T_v) \hat{\hat{v}}_{2,k} + (\tau_v T_v) \dot{\hat{v}}_{2,k} \quad \text{with} \quad \hat{v}_{2,0} = 0
\]

(16)

\[
\dot{\hat{u}}_{k+1} = (1 - \tau_u T_u) \hat{\hat{u}}_{k} + (\tau_u T_u) \dot{\hat{u}}_{k} \quad \text{with} \quad \hat{u}_{0} = 0
\]

(17)

where \(\tau_v\) and \(\tau_u\) denote the corresponding parameter of \(H(S)\). The used values of these variables and the other parameters of the implementation are tabulated in Table 2. For each one of the motors in a single module, a dedicated FLC is utilized, the membership functions are kept static and only the parameters of the defuzzifiers (\(\phi\)) are adjusted.

Initially, the motors are at rest, i.e. at zero angle and at zero angular velocity for each, and the desired angular position for all three of them has been set as \(\pi/2\) radians. The rationale behind this selection is due to the philosophy of \(I\)-Cubes project, which presumes that each actuator performs \(\pi/2\) radians [30-31], so that at a larger scale, the motion planning algorithm could exploit the information coming from the lowest level of the design, and this is strictly dependent upon the control performance. Although the motion of multiple modules is possible, the application of \(I\)-Cubes constrains the link level motion to one-at-a-time basis for each one of the motors. In order to demonstrate the performance of the proposed control scheme, we test the scheme while all three motors are actuated. By this means, the concept presented in the previous section could fairly be assessed.

In the implementation stage, we observed that the gearing illustrated in Fig. 1 introduces

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**Fig. 7** Extracted control signal errors, phase space behaviors and parametric evolutions

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nonlinear friction terms and the angular velocity of the motors even without the link load changes under the same input voltage. Furthermore, again because of the inevitable gearing of the design, there exists a considerable dead zone in the motor actuation. Approximately between \( \pm 0.8 \) volts of average control input magnitude, the motors do not respond at all.

These difficulties apparently require an intelligent controller meeting the desired performance specifications. In Fig. 6, the tracking errors and controller outputs for the base, shoulder and elbow motors are illustrated in the units of radians, radians/second and volts respectively. Parallel to what the presented design prescribes, clearly from the first two rows, the error in angular positions and velocities tend to zero. The bottom row depicts the evaluated control signals. As mentioned before, the existing dead zone in the system dynamics for all three links causes an oscillatory behavior in the applied motor voltages. When a tiny nonzero error is measured, the controller tries to improve it, however, this requires the excess of the dead zone level. If this causes a nonzero error having the opposite sign, the procedure repeats. Therefore, an oscillatory behavior is observed in the control signals. The performance in terms of maintaining the precision with the presence of such a nonlinearity is satisfactory and the smoothness of the control signal is another important observation that should be highlighted.

Fig. 7 illustrates the values of \( s_c \) postulated in Proposition 2, the behavior in the corresponding phase space defined for each motor, and the evolution of the adjustable parameter vector norms of the three FLCs. For the parametric evolution, we plot the value of \( \| \phi_c \| = (\phi_c^T \phi_c)^{1/2} \), which is an appropriate measure for evaluating the stability of the tuning mechanism. As claimed in Theorem 5, the trends of the evolutions seen in the bottom subplots suggest that the excited parameters are tuned until the goal described for each one of the motors is reached. The extracted values of the control discrepancies converge to zero, and the tuning becomes inactive during the sliding mode.

In order to avoid adverse effects of the sign function appearing in the reaching law and the tuning law, \( \text{sgn}(s) = s/|s| \) approximation is used with \( \delta = 0.25 \). This significantly reduces the chattering magnitudes due to the discontinuity of the original sign function, and helps the smoothly inactivation of the tuning scheme in the vicinity of the sliding manifold.

For the presented results, for all three FLCs, there exists a \( 0 < \Lambda_1 < 0.92 \) and \( 0 < \Lambda_2 < 0.8375 \) enabling the existence of a strictly positive \( \Gamma \) satisfying the inequality \( 0 < \Gamma < 0.2418 \). Apparently, the numbers obtained justify the theoretical background discussed in the third section.

The computational burden of the proposed strategy for each FLC and its tuning mechanism requires 113 floating point operations and 12 comparisons per control period. This result is in good compliance with the needs of the modularity concept of distributed robotics.

5. CONCLUSIONS

This paper discusses the issues related to real-time adaptive fuzzy sliding mode control of a class of bipartite modular robotic systems called I-Cubes. Since one of the core issues in the design and implementation of the I-Cubes entity is to keep the modularity with cheap components, the control precision becomes a tedious objective due to the time-sparse observations of data, numerical differentiations, noise, nonlinear friction effects, dead zone effects and limited mobility in applicable voltage range for the motors used. In order to address all these difficulties in an appropriate manner, we suggest the use of adaptive fuzzy sliding mode control scheme with triangular and stationary membership functions. The defuzzifier parameters of the FLCs are adjusted according to a rule that drives the corresponding subsystem dynamics into a predefined sliding regime. The analysis of the approach has been presented to address the issues described in continuous time and discrete time, and the stability conditions are derived. The results observed justify the theoretical claims, both in terms of performance in operational space and in terms of the stability in adjustable parameter space. The mentioned difficulties are alleviated through the use of the proposed adaptation mechanism by assuming that the plant under control belongs to a certain class, and with significantly low computational.
burden. When the addressed difficulties are considered with what have been obtained, it can fairly be claimed that the proposed technique constitutes a good solution for admissibly precise control with such cheap hardware, and is therefore a good candidate for control applications in modular robotics.

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Adaptive Fuzzy Sliding Mode Control For A Class Of Bipartite Modular Robotic Systems

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