POSITION CONTROL OF SYNCHRONOUS MACHINE USING SLIDING MODE TECHNIQUES

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ABSTRACT

The sliding mode controller is designed for a class of non linear dynamic systems to tackle the problems with model uncertainties, parameter fluctuations and external disturbances. By this design, the bounds of the uncertainties are not required to be known in advance. In this paper, we develop a sliding mode controller for the robust position control of synchronous motor. Also, the load torque disturbance affects directly the motor shaft. An observer is considered to overcome the problem of torque disturbance. An asymptotically stable observer gain can be obtained without affecting the overall system response. The simulation results show the effectiveness of the proposed control strategy with desired tracking accuracy and robustness.

Keywords: synchronous machine, sliding mode control, torque observer, position control.

1. INTRODUCTION

The control of the synchronous machine (SM) must take into account machine specificities: the high order of the model, the nonlinear functioning as well as the coupling between the different variables of control. Furthermore, the machine parameters depend generally on the operating point and vary either on the temperature (resistance), or with the magnetic state of the synchronous machine. These parametric variations modify the performances of the control system when we use a regulator or a control law with fixed parameters.

Since the work of V. I. Utkin proposed in 1977 [1], significant interest on variable structure systems (VSS) and sliding mode control (SMC) has been generated in the control research community worldwide. The variable structure control (VSC) possesses high robustness using the sliding mode control that can offer many good properties such as good performance against unmodelled dynamics, insensitivity to parameter variation, complete rejection of disturbances, and fast dynamic [2].

Variable structure control with sliding mode, is one of the effective non linear robust control approaches since it provides system dynamics with an invariance property to uncertainties once the system dynamics reach the sliding surface [1, 3, 4]. The main disadvantage of this approach is the high switching frequency of the control action or chattering that VSC system exhibit. Chattering is undesirable since it can excite the unmodeled high frequency dynamics in the non linear system control. Introducing a boundary layer is one of the most common techniques
used, with the cost of an important degradation in tracking performance [5].

The new industrial applications necessitate speed variators having high dynamics performances, a good precision in permanent regime, and a high capacity of overload on all range of speed/position and a robustness to different perturbations. Thus, the recourse to robust control algorithms is desirable in stabilization and in tracking trajectories. The variable structure control possesses this robustness using the sliding mode control that can offer high performances against internal and external disturbance. These advantages of sliding mode control can be employed in the position and speed control of an alternative current servo system [1, 6-8].

In this paper the application of sliding mode control in synchronous position control is described. The organization of this work is as follows: first, the vector control principle for synchronous motor drive and model of the voltage source inverter are presented; next, the proposed controller is described, and used to control the position synchronous motor, and by the way, a torque observer is developed. Simulation results are given to show the effectiveness of this controller and finally conclusions are summarized in the last section.

2. DYNAMİC MODEL OF SYNCHRONOUS MOTOR

The dynamic model of synchronous motor in d-q frame can be represented by the following equations [9, 10]:

\[
\begin{align*}
\Omega &= \frac{d}{dt} \theta, \quad \theta = \int \Omega \, dt, \quad \omega = \frac{d}{dt} \theta_c = P \Omega, \\
\theta_c &= P \theta.
\end{align*}
\]

The flux linkage equations are:

\[
\begin{align*}
\phi_{ds} &= L_{ds} i_{ds} + M_{fd} i_f \\
\phi_{qs} &= L_{qs} i_{qs} \\
\phi_f &= L_f i_f + M_{fd} i_{ds}
\end{align*}
\]

Where \( R_s \) – stator resistance, \( R_f \) – field resistance, \( L_{ds}, L_{qs} \) – respectively direct and quadrature stator inductances, \( L_f \) – field leakage inductance, \( M_{fd} \) – mutual inductance between inductor and armature, \( \phi_{ds} \) and \( \phi_{qs} \) – respectively direct and quadrature flux, \( \phi_f \) – field flux, \( T_e \) – electromagnetic torque, \( T_i \) – external load disturbance, \( P \) – pair number of poles, \( B \) – is the damping coefficient, \( J \) – is the moment of inertia, \( \omega \) – electrical angular speed of motor, \( \Omega \) – mechanical angular speed of motor, \( \theta \) – mechanical rotor position, \( \theta_c \) – electrical rotor position.

3. VOLTAGE SOURCE İNVERTER

The power circuit of a three-phase bridge inverter using six switch device is shown in figure 1. The dc supply is normally obtained from a utility power supply through a bridge rectifier and LC filter to establish a stiff dc voltage source [11].

Fig. 1. Voltage inverter

The switch \( T_{ci} \) \( (c \in \{1, 2, 3\}, i \in \{1, 2\}) \) is supposed perfect. The simple inverter voltage can be presented by logical function connexion in matrix form as [11].
where the logical function connexion \( F_{c1} \) is defined as: \( F_{c1} = 1 \) if the switch \( T_{c1} \) is closed, \( F_{c1} = 0 \) if the switch \( T_{c1} \) is opened, \( U_c \) is the voltage feed inverter.

### 4. DESCRIPTION OF THE SYSTEM

The schematic diagram of the speed control system under study is shown in figure 2. The field current \( i_f \) of the synchronous machine, which determines the field flux level is controlled by voltage \( V_f \). The parameters of the synchronous machine are given in the Appendix.

The self-control operation of the inverter-fed synchronous machine results in a rotor field oriented control of the torque and flux in the machine. The principle is to maintain the armature flux and the field flux in an orthogonal or decoupled axis. The flux in the machine is controlled independently by the field winding and the torque is affected by the fundamental component of armature current \( i_{qs} \). In order to have an optimal functioning, the direct current \( i_{ds} \) is maintained equal to zero \([10, 12]\).

Substituting (4) in (3), the electromagnetic torque can be rewritten for \( i_f = \text{constant} \) and \( i_{ds} = 0 \) as follow:

\[
T_e(t) = \lambda i_{qs}(t)
\]

where \( \lambda = pM_{fa}i_f \).

In the same conditions, it appears that the \( V_{ds} \) and \( V_{qs} \) equations are coupled. We have to introduce a decoupling system, by introducing the compensation terms \( emf_d \) and \( emf_q \) in which

\[
emf_d = \omega L_{qs}i_{qs},
\]

\[
emf_q = -\omega L_{ds}i_{ds} - \omega M_{af}i_f.
\]

Figure (2) shows the proposed schematic diagram of the position control of the synchronous motor using sliding mode control.

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**Fig.2.** System configuration of field-oriented synchronous motor.
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The blocks SMC_{ds}, SMC_{ids} et SMC_{iqs} are regulators, the first is the sliding mode controller for position, the second is the sliding mode regulator for the direct current and the third is the sliding mode regulator for the quadrature current. The load torque is estimated by the “Load torque observer”. To avoid the appearance of an inadmissible value of current, a saturation bloc is used.

5. SLIDING MODE CONTROL

Consider a nonlinear system which can be represented by the following state space model in a canonical form [3]:

\[ x^{(n)}(t) = f(x(t), t) + g(x(t), t)u + d(t) \]

\[ y(t) = x(t) \] (8)

Where \( x = [x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)]^T \) is the state vector, \( f(x(t), t) \) and \( g(x(t), t) \) are nonlinear functions, \( u \) is the control input, \( d(t) \) is the external disturbances. The objective of the control is to determine a control law \( u(t) \) to force the system output \( y(t) \) in (8) to follow a given bounded reference signal \( y_d(t) \), that is, the tracking error \( e(t) = y_d(t) - y(t) \) and its forward shifted values, defined as

\[ e^{(i)}(t) = y_d^{(i)}(t) - y^{(i)}(t) \]

\[ = x_d^{(i)}(t) - x^{(i)}(t), \] (9)

should be small. The design of SMC involves two tasks. The first one is to select the switching hyperplane to prescribe the desired dynamic characteristics of the controlled system. The second one is to design the discontinuous control such that the system enters the sliding mode and remains in it forever [3, 13].

In this paper, we use the sliding surface proposed par J.J. Slotine, \( s(x, t) = \left( \frac{d}{dt} + \lambda \right)^{n-1} e(t) \) (10) in which \( e = x_d(t) - x(t) \), \( \lambda \) is a positive coefficient, and \( n \) is the system order. It remains to be shown that the control law can be constructed so that the sliding surface will be reached. Then, a sliding hyperplane can be represented as \( s(x, t) = 0 \).

Consider a Lyapunov function:

\[ V = \frac{1}{2} s^2 \] (11)

From Lyapunov theorem we know that if \( \dot{V} \) is negative definite, the system trajectory will be driven and attracted toward the sliding surface and remain sliding on it until the origin is reached asymptotically [6]:

\[ \dot{V} = s \dot{s} \] (12)

The simplified 1st order problem of keeping the scalar \( s(x, t) \) at zero can be achieved by choosing the control law \( u(t) \). A sufficient condition for the stability of the system is

\[ \frac{1}{2} \frac{d}{dt} s^2 \leq -\eta |s| \] (13)

where \( \eta \) is a positive constant.

The equation (13) is called reaching condition or sliding condition. \( s(t) \) verifying (13) is referred to as sliding surface, and the system’s behaviour once on the surface is called sliding mode. If the control input is so designed that the inequality (13) is satisfied, together with the properly chosen sliding hyperplane, the state will be driven toward the origin of the state space along the sliding hyperplane from any given initial state. This is the way of the SMC that guarantees asymptotic stability of the systems.

The process of sliding mode control can be divided in two phases, that is, the approaching phase and the sliding phase. The sliding mode control law \( u(t) \) consists of two terms, equivalent term \( u_{eq}(t) \), and switching term \( u^s(t) \).

In the sliding phase, where \( s(x, t) = 0 \) and \( \dot{s}(x, t) = 0 \), the equivalent term \( u_{eq}(t) \) is designed to keep the system on the sliding surface. In the approaching phase, where \( s(x, t) \neq 0 \), the switching term \( u^s(t) \) is designed to satisfy the reaching condition (13).

While in sliding phase we have:

\[ s(x, t) = 0 \] (14)

By solving the above equation formally for the control input, we obtain an expression for \( u \) called the equivalent control \( u_{eq} \), which can be interpreted as the continuous control law that would maintain \( s(x, t) = 0 \) if the dynamics were exactly known.

In order to satisfy sliding conditions (13) and to despite uncertainties on the dynamic of the system, we add a discontinuous term across the
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surface \( s(x,t) = 0 \), so the sliding mode control law \( u(t) \) has the following form:
\[
u = u^e + u^n
\]
where \( K_f \) is the control gain.

For a defined function \( \phi \):
\[
\text{sat}(\phi) = \begin{cases} 
1, & \text{if } |\phi| \geq 1 \\
\phi, & \text{if } |\phi| < 1
\end{cases}
\]
(16)

The controller described by the equation (15) presents high robustness, insensitive to parameter fluctuations and disturbances [1, 3, 4, 14, 15], but it will have high-frequency switching (chattering phenomena) near the sliding surface due to \( \text{sgn} \) function involved. These drastic changes of input can be avoided by introducing a boundary layer with width \( \varepsilon \) [3, 4, 15]. Thus replacing \( \text{sgn}(s(t)) \) by \( \text{sat}(s(t)/\varepsilon) \) in (15), we have
\[
u = u^e - K_f \text{sat}(s(x,t))
\]
(17)

Where
\[
\varepsilon > 0,
\]
\[
\text{sat}(\phi) = \begin{cases} 
\text{sgn}(\phi), & \text{if } |\phi| \geq 1 \\
\phi, & \text{if } |\phi| < 1
\end{cases}
\]

5.1. POSITION CONTROL
The position error is defined by:
\[
e = \theta_{ref} - \theta
\]
(18)
For \( n=2 \), the position control manifold equation can be obtained from equation (10) as follow:
\[
s(e) = \lambda_0 e + \frac{d}{dt} e
\]
(19)
The equation of the motion (2) can be rewritten:
\[
\dot{\theta} = -\frac{B}{J} \Omega + \frac{p \lambda}{J} i_{qs} - \frac{p T_l}{J}
\]
(20)
\[
s(\theta) = \lambda_0 \dot{\theta} - \lambda_0 \dot{\theta}_{ref} + \dot{\theta}_{ref} - \dot{\theta}
\]
(21)
\[
\dot{s}(\theta) = \lambda_0 \dot{\theta}_{ref} + \dot{\theta}_{ref} + \left( \frac{B}{J} - \lambda_0 \right) \dot{\theta} + \frac{p T_l}{J} - \frac{p \lambda}{J} i_{qs}
\]
(22)
During the sliding mode and in permanent regime, we have
\[
s(\theta) = 0, \ \dot{s}(\theta) = 0, \ i^n_{qs} = 0
\]
The current control \( i_{qs} \) is defined by:
\[
i_{qs} = i_{qs}^e + i^n_{qs}
\]
(23)
In which:
\[
i_{qs}^e = \frac{J}{p \lambda} \left( \lambda_0 \dot{\theta}_{ref} + \dot{\theta}_{ref} + \left( \frac{B}{J} - \lambda_0 \right) \dot{\theta} + \frac{p T_l}{J} \right)
\]
(24)
\[
i^n_{qs} = K_o \text{sgn}(s(\Omega))
\]
(25)
\( K_o \)– positive constant.

5.2. DIRECT CURRENT CONTROLLER
The direct current error is defined by:
\[
e_d = i_{ds\_ref} - i_{ds}
\]
(26)
For \( n=1 \), the direct current control manifold equation can be obtained by:
\[
s(i_{ds}) = i_{ds\_ref} - i_{ds}
\]
(27)
Substituting the expression of \( i_{ds} \) given by equation (1) and (4) in equation (27) we obtain:
\[
s(i_{ds}) = \frac{d}{dt} i_{ds\_ref} + \frac{R_s}{L_{ds}} i_{ds} - \frac{L_{qs}}{L_{ds}} i_{qs} \omega - \frac{1}{L_{qs}} v_{ds}
\]
(28)
During the sliding mode and in permanent regime, we have
\[
s(i_{ds}) = 0, \ \dot{s}(i_{ds}) = 0, \ i^n_{ds} = 0
\]
The control voltage \( v_{ds\_ref} \) is defined by:
\[
v_{ds\_ref} = v_{ds}^e + v^n_{ds}
\]
(29)
Where:
\[
v_{ds}^e = \left( \frac{d}{dt} i_{ds\_ref} + \frac{R_s}{L_{ds}} i_{ds} - \frac{L_{qs}}{L_{ds}} i_{qs} \omega \right) L_{ds}
\]
(30)
\[
v^n_{ds} = K_d \text{sgn}(s(i_{ds}))
\]
(31)
\( K_d \)– positive constant.

5.3. QUADRATURE CURRENT CONTROL
The quadrature current error is defined by:
\[
e_q = i_{qs\_ref} - i_{qs}
\]
(32)
For \( n=1 \), the quadrature current control manifold equation can be obtained by:
\[
s(i_{qs}) = i_{qs\_ref} - i_{qs}
\]
(33)
Then, we have
\[ \dot{s}(i_{qs}) = i_{qs_{ref}} - i_{qs} \]  
(34)

Substituting the expression of \( i_{qs} \) given by equation (1) and (4) in equation (34) we obtain:
\[ \dot{s}(i_{qs}) = \frac{d}{dt} i_{qs_{ref}} + \frac{R}{L_{qs}} i_{qs} + p \frac{L_{ds}}{L_{qs}} \alpha_{ds} + M_{jd} \alpha_{j} - \frac{1}{L_{qs}} v_{qs} \]  
(35)

During the sliding mode and in permanent regime, we have
\[ \dot{s}(i_{qs}) = 0, \dot{i}(i_{qs}) = 0, i_{qs} = 0 \]

The control voltage \( v_{qs_{ref}} \) is defined by:
\[ v_{qs_{ref}} = v_{qs}^{eq} + v_{qs}^{n} \]  
(36)

Where:
\[ v_{qs}^{eq} = K_q \, \text{sign}(s(i_{qs})) \]  
(37)

\[ v_{qs}^{n} = K_q \, \text{sign}(s(i_{qs})) \]  
(38)

\( K_q \) - positive constant.

6. LOAD TORQUE OBSERVER

The motion equation of the synchronous motor (2) can be expressed in state space as follows:
\[ \dot{X} = A \, X + B \, u \]  
(39)

Where \( X = \begin{bmatrix} \Omega \\ T_i \end{bmatrix} \), \( u = i_{qs} \),
\[ A = \begin{bmatrix} -\frac{B}{J} & -\frac{1}{J} \\ \frac{J}{L} & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}. \]

It is well known that observer is available when input is unknown and inaccessible. For simplicity a 0-observer is selected. In this paper, the load torque \( T_i \) is estimated by using an observer. A linear asymptotic observer is designed in the same form as the original system (39) with an additional input depending on the mismatch between the real values and the estimated values of the output vector [16, 17]. The system equation can be expressed as:
\[ \dot{\hat{X}} = A \, \hat{X} + B \, u + L \, (Y - C \, \hat{X}) \]  
(40)

Where \( L = \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \)

Where \( \hat{X} \) is an estimate of the system state vector, and \( L \) is the proportional gain vector to be chosen so as to achieve prespecified error characteristics.

The motion equation with respect to mismatch \( \tilde{X} = \hat{X} - X \) is of form
\[ \dot{\tilde{X}} = (A - LC) \tilde{X} \]  
(41)

The behavior of the mismatch governed by homogeneous equation is determined by the eigenvalues of the matrix \((A+LC)\). For the observable system they may be assigned arbitrarily by a proper choice of the gain vector \( L \). It means that any desired rate of convergence of the mismatch to zero or estimate \( \hat{X}(t) \) to the state vector \( X(t) \) may be provided. To ensure that the observer is stable the instantaneous eigenvalues of the observer have to be placed in the left half side plane. The characteristics equation is given by:
\[ \text{Det} \left[ p \, I - (A - LC) \right] = 0 \]  
(42)

Where \( I \) is the identity vector, and \( p \) is the Laplace operator.

The gain vector \( L \) is defined by imposing the poles in the characteristics equation.

7. SIMULATION AND RESULTS

In order to validate the control strategies as discussed above, digital simulation studies were made the system described in figure 2. The position and currents loops of the drive were also designed and simulated respectively with sliding mode control. The feedback control algorithms were iterated until best simulation results were obtained.

The position loop was closed, and transient response was tested with both current controller and position control. The simulation of the starting mode without load is done, followed by reversing of the reference \( \theta_{ref} = \pm 3 \text{rad/s} \) at \( t_3 = 2 \text{s} \).

The load \( (T_i) \) is applied in two period:

The reference \( \theta_{ref} = +3 \text{rad} \), the load \( (T_i = +8 \text{Nm}) \) is applied at \( t_1 = 1 \text{s} \) and eliminated at \( t_2 = 1.5 \text{s} \).

The reference \( \theta_{ref} = -3 \text{rad} \), the load \( (T_i = -8 \text{Nm}) \) is applied at \( t_4 = 3 \text{s} \) and eliminated at \( t_5 = 3.5 \text{s} \).
The simulation is realized using the SIMULINK software in MATLAB environment. Figure 3 shows the performances of the sliding mode controller using the load torque observer.

The control presents the best performances, to achieve tracking of the desired trajectory. The sliding mode controller rejects the load disturbance rapidly with no overshoot and with a negligible steady state error. The current is limited in its maximal admissible value by a saturation function. The decoupling of torque-flux is maintained in permanent mode.

7.1. ROBUSTNESS TESTS
In order to test the robustness of the used method we have studied the effect of the parameters.
uncertainties on the performances of the position control.

To show the effect of the parameters uncertainties, we have simulated the system with different values of the parameter considered and compared to nominal value (real value). Three cases are considered:
1. The moment of inertia (±50%).
2. The stator and rotor resistances (+50%).
3. The stator and rotor inductances (+20%).

To illustrate the performances of control, we have simulated the starting mode of the motor without load, and the application of the load ($T_L = +8 \text{Nm}$) at the instance $t_1 = 1 \text{s}$ and its elimination at $t_2 = 2 \text{s}$; in presence of the variation of parameters considered (the moment of inertia, the stator resistances, the stator inductances) with position step of $+3 \text{rad/s}$.

Figure (4) shows the tests of robustness realized with sliding mode control for different values of the moment of inertia.

Figure (5) shows the tests of robustness realized with sliding mode control for different values of stator resistances.

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**Fig. 4.** Test of robustness for different values of the moment of inertia: 1) – 50%, 2) nominal case, 3) +50%.

**Fig. 5.** Test of robustness for different values of stator resistances: 1) nominal case, 2) +50%.
Figure (6) shows the tests of robustness realized with sliding mode control for different values of stator inductances.

The results of figures (4, 5, 6) show a decrease or increase of the moment of inertia $J$, the resistances or the inductances doesn’t have any effects on the performances of the technique used (figure 5 and 6). An increase of the moment of inertia gives best performances, but it presents a slow dynamic response (figure 4). The proposed control gives to our controller a great place towards the control of the system with unknown parameters.

8. CONCLUSIONS
In this study, hydroxyproline levels in the skin of A sliding mode control method has been proposed and used for the control of a synchronous machine using field oriented control. A simple observer of the load torque is presented. Simulation Results show good performances obtained with proposed control. It has been shown the robustness of proposed control in relation to the presence internal and external perturbations. With a good choice of parameters of control and the smoothing out control discontinuity, the chattering effects are reduced, and the torque fluctuations are decreased. The position control operates with enough stability and has strong robustness to parameter variations. Furthermore, this regulation presents a simple robust control algorithm that has the advantage to be easily implantable in calculator.

APPENDIX
Three phases SM parameters:
Rated output power 3HP, Rated phase voltage 60V, Rated phase current 14 A, Rated field voltage $v_f=1.5V$, Rated field current $i_f=30A$, Stator resistance $R_s=0.325\Omega$, Field resistance $R_f=0.05\Omega$, Direct stator inductance $L_{ds}=8.4\text{ mH}$, Quadrature stator inductance $L_{qs}=3.5\text{ mH}$, Field leakage inductance $L_f=8.1\text{ mH}$, Mutual inductance between inductor and armature $M_{fd}=7.56\text{ mH}$, The damping coefficient $B=0.005\text{ N.m/s}$, The moment of inertia $J=0.05\text{ kg.m}^2$, Pair number of poles $p=2$.

REFERENCES

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